Kinematika Robot Beroda

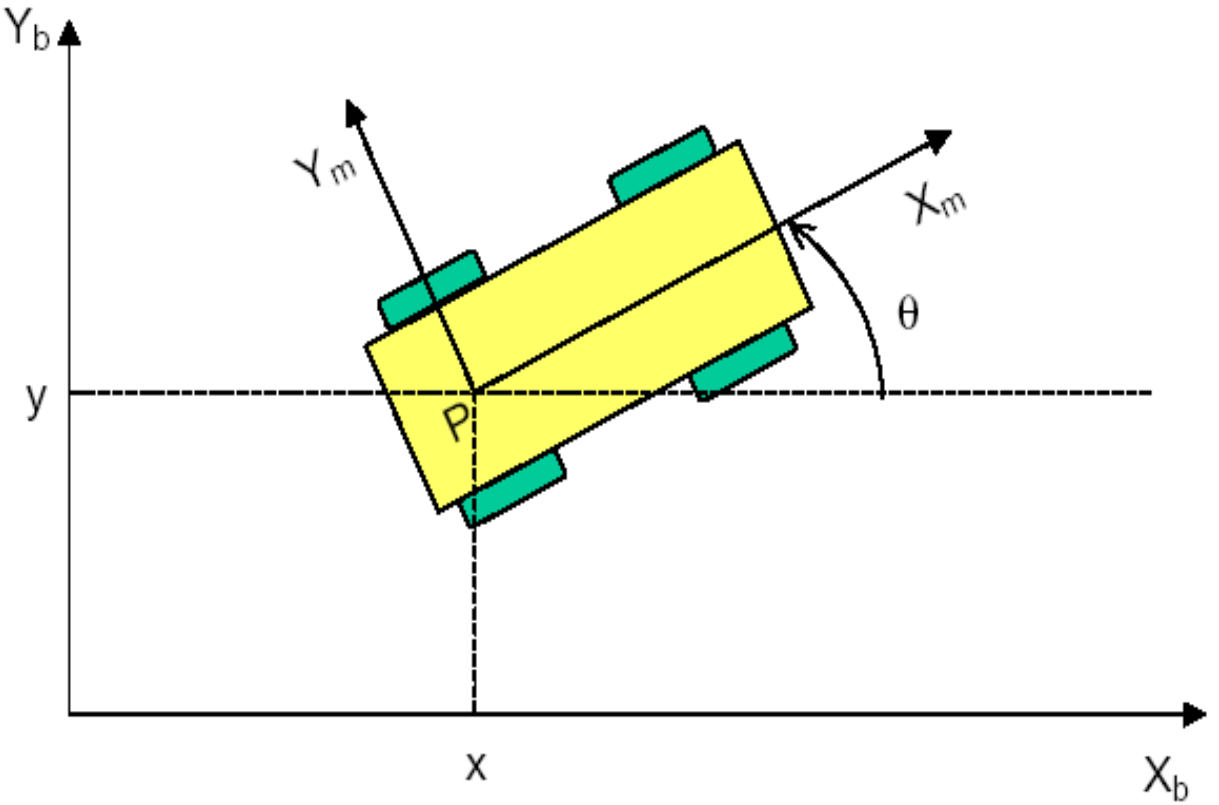
Eko Didik Widianto

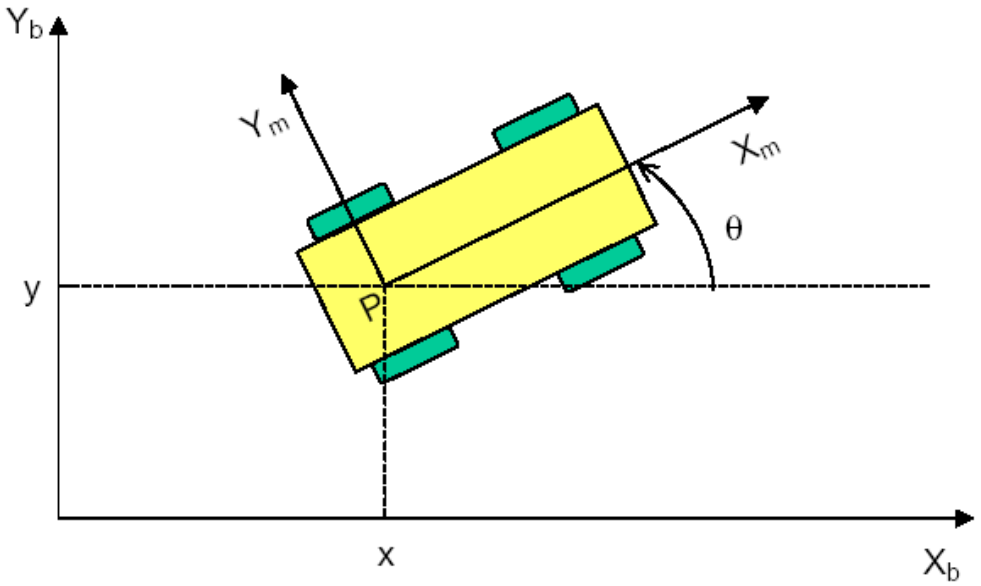
TKC225 - Robotika

PROGRAM STUDI SISTEM KOMPUTER
UNIVERSITAS DIPONEGORO

Posisi dan Gerak Robot

- Robot bergerak ke posisi (x,y) dengan sudut θ. Berapa jarak yang ditempuh robot?
- Robot bergerak sejauh Xm dengan sudut θ. Dimana posisi robot sekarang?





Notasi

Posture: position(x, y) and orientation ☐

- $\{X_m, Y_m\}$ – moving frame
- $\{X_b, Y_b\}$ – base frame

- Posture matrix
- Rotation matrix
- Base frame matrix
- Moving frame matrix
- Inverse rotation

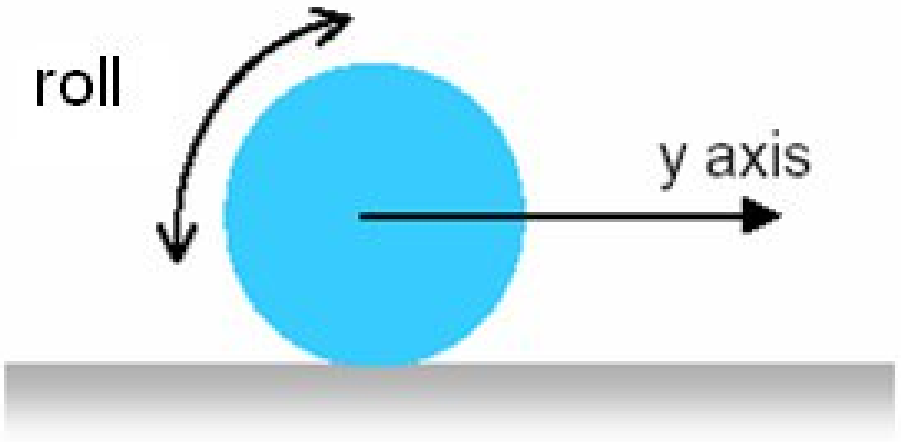
$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

robot posture in base frame

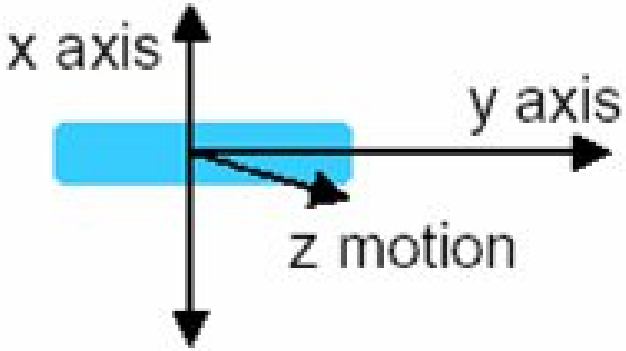
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation matrix expressing the orientation of the base frame with respect to the moving frame

Roda (Wheel)



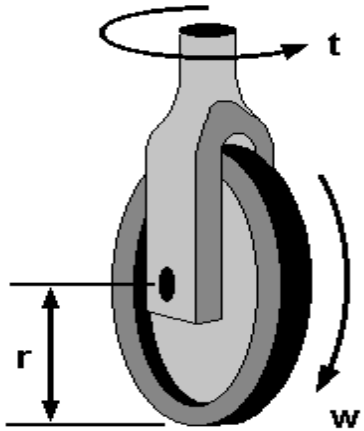
Rolling motion

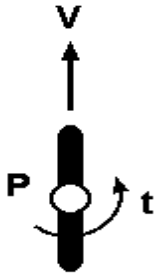


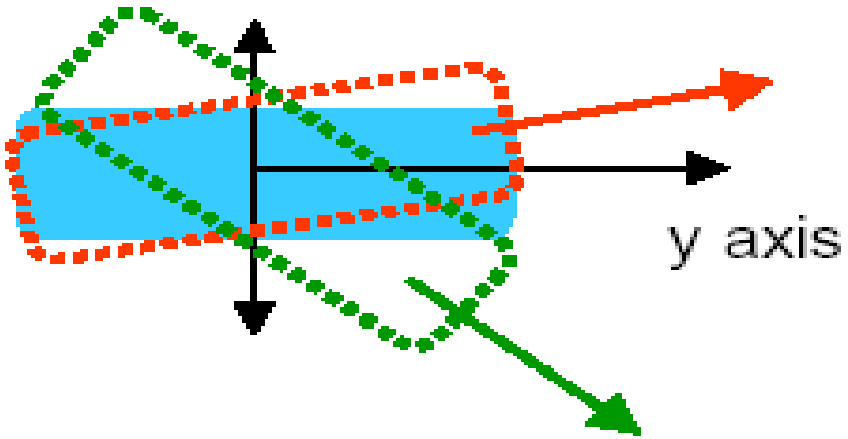
Lateral slip

Steered Wheel

- **Steered wheel**
  - Orientasi dari sumbu rotasi roda dapat dikendalikan





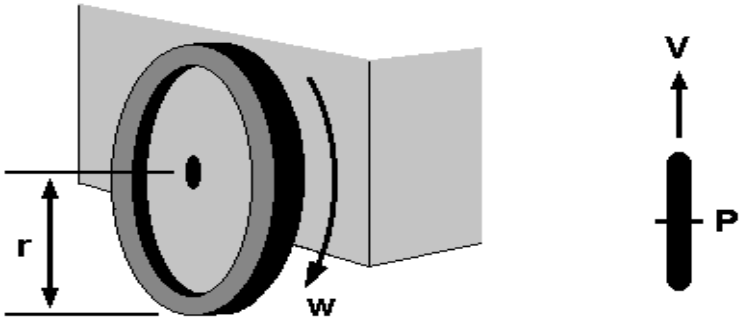


Parameter Roda

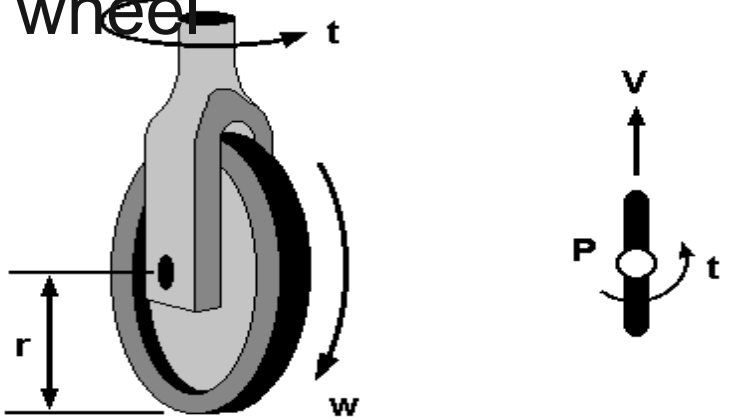
- Parameter Roda:
  - r = Jari2 Roda
  - v = Kecepatan Linier Roda
  - w = Kecepatan sudut roda
  - t = Kecepatan pengemudian (steering)

Jenis Roda

Fixed wheel

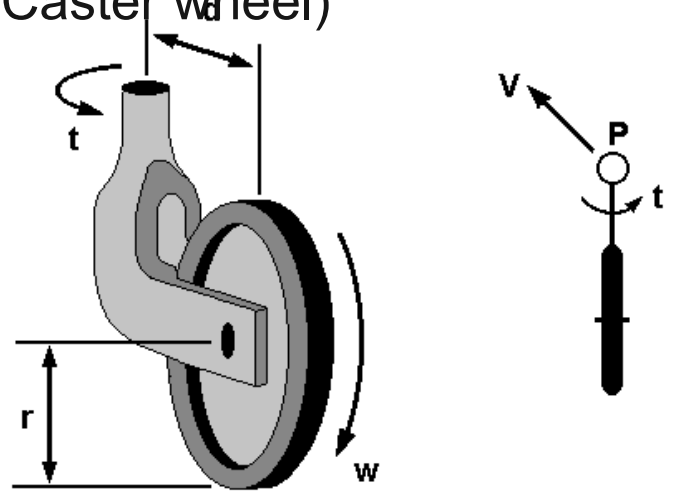


Centered orientable wheel

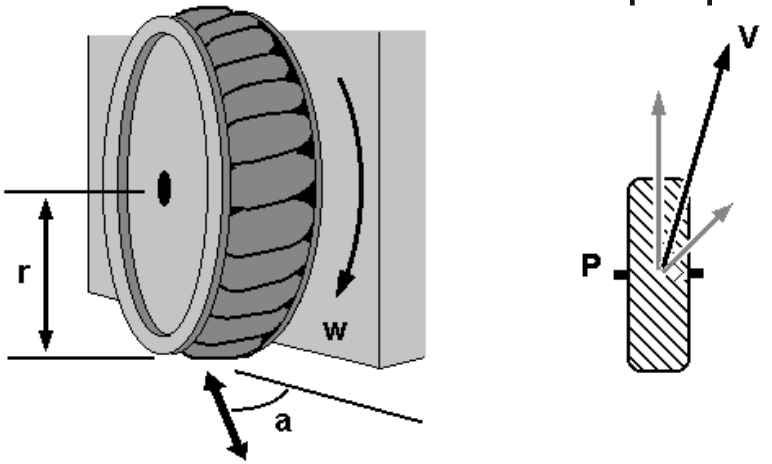


Off-centered orientable wheel

(Caster wheel)



Swedish wheel: omnidirectional property



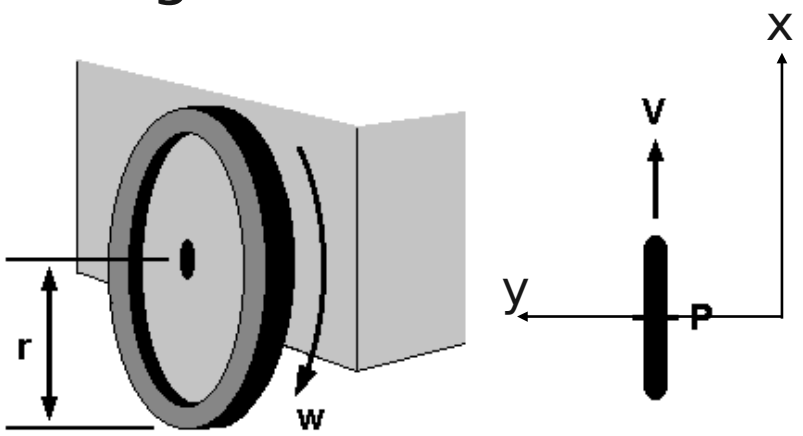
Fixed wheel

- Kecepatan Titik **P**

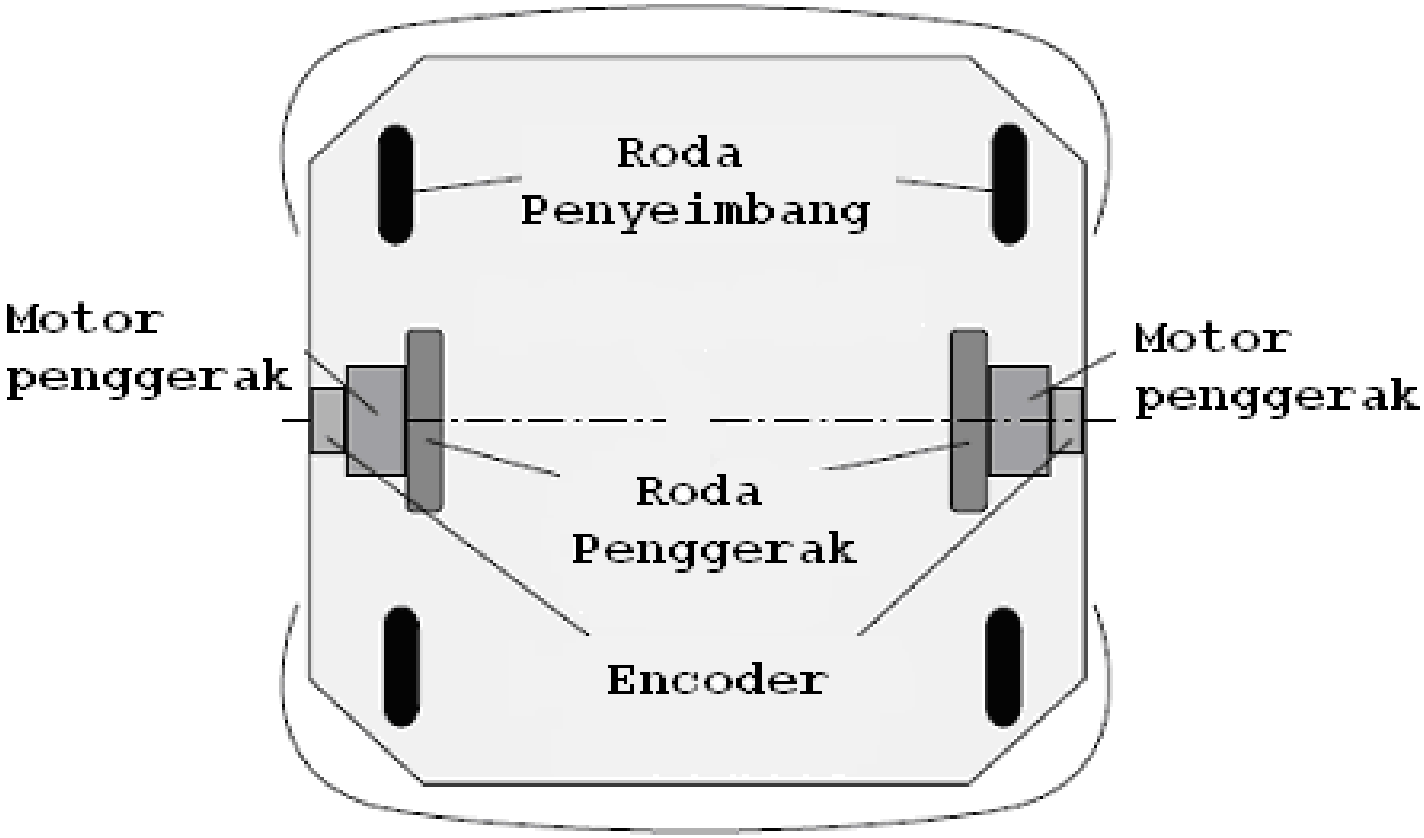
$$V = (r \times w)a_x$$

dimana, a_x : vektor satuan arah sumbu X

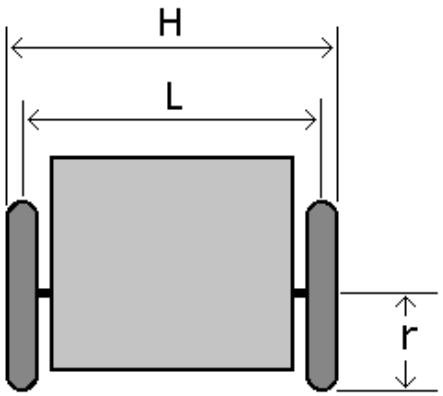
- Titik P tidak dapat berpindah menuju arah tegak lurus bidang roda

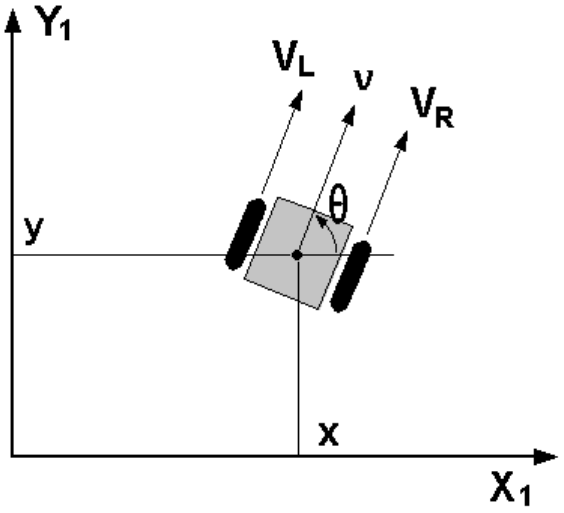


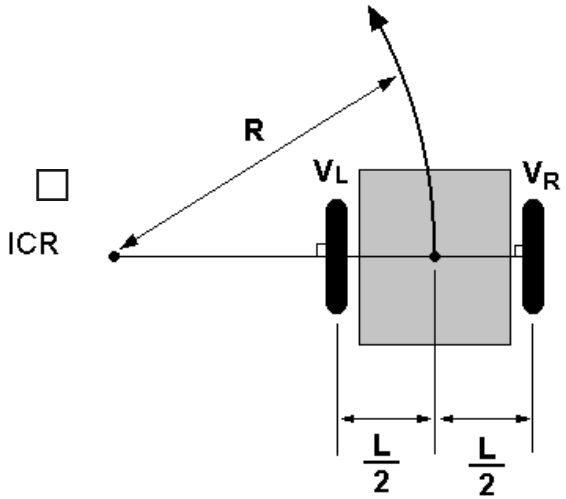
Arsitektur Mobot jenis kemudi diferensial



Differential Drive







- Posture robot

$$P = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

(x,y) : Posisi robot
θ : Orientasi robot

- Control Input

$$U = \begin{pmatrix} v \\ w \end{pmatrix}$$

v : Kecepatan linier **robot**
w : Kecepatan sudut **robot**

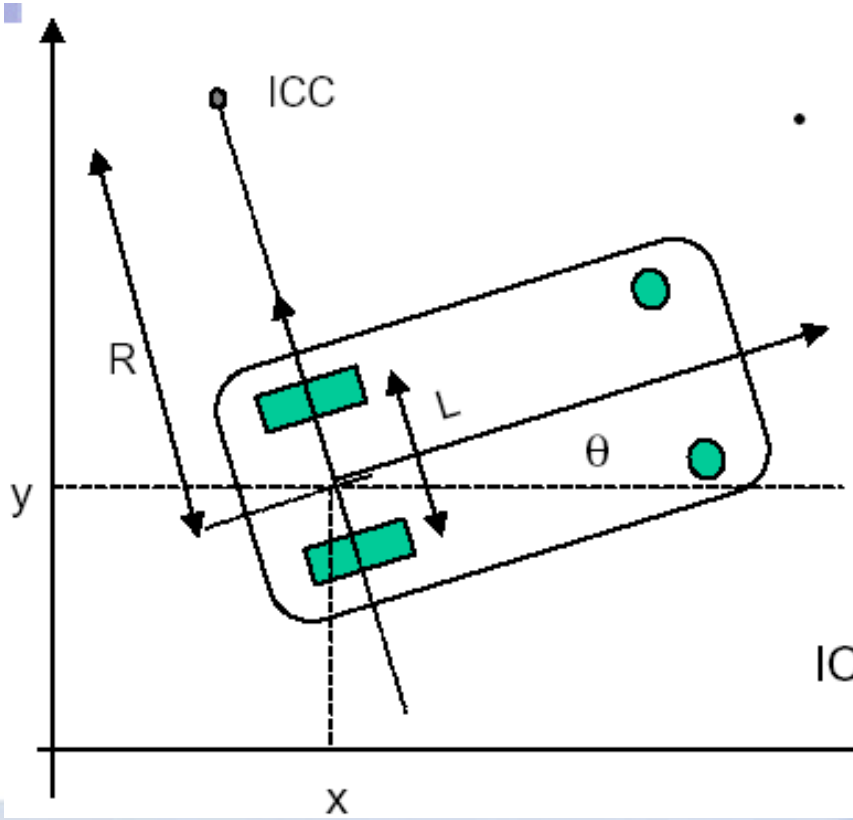
Differential Drive

$V_R(t)$ – Kecepatan Linier roda kanan

$V_L(t)$ – Kecepatan Linier roda kiri

r – Jari2 nominal masing2 roda

R – instantaneous curvature radius trayektori robot (Jarak dari ICC ke titik tengah antara dua roda).



- 2 drive rolling wheels

  Property: pada setiap saat, roda kiri dan kanan harus mengikuti trayektori disekitar ICC dengan kecepatan sudut yang sama

$$\omega(R - \frac{L}{2}) = V_R \qquad \omega(R + \frac{L}{2}) = V_L$$

$$ICC = (x - R\sin\theta, y + R\cos\theta)$$

Differential Drive

Model Posture : Model kinematik terhadap bumi (bid. kartesian)

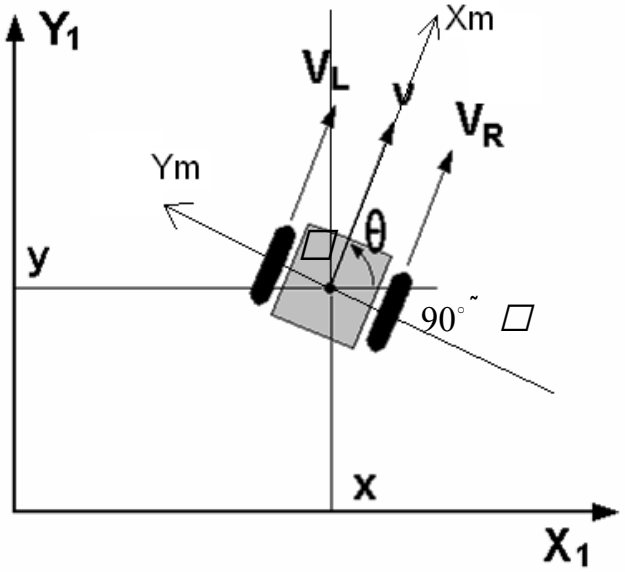
- Hubungan antara input kendali dengan kecepatan roda

$$V_L = r\,\omega_L \qquad V_R = r\,\omega_R$$

$$\omega = \frac{V_R - V_L}{L} \qquad v = \frac{V_R + V_L}{2}$$

- Persamaan Kinematik

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$



- Sistem Nonholonomic

$$\sin\theta \quad -\cos\theta \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

Kecepatannya tidak dapat diturunkan dari posisi

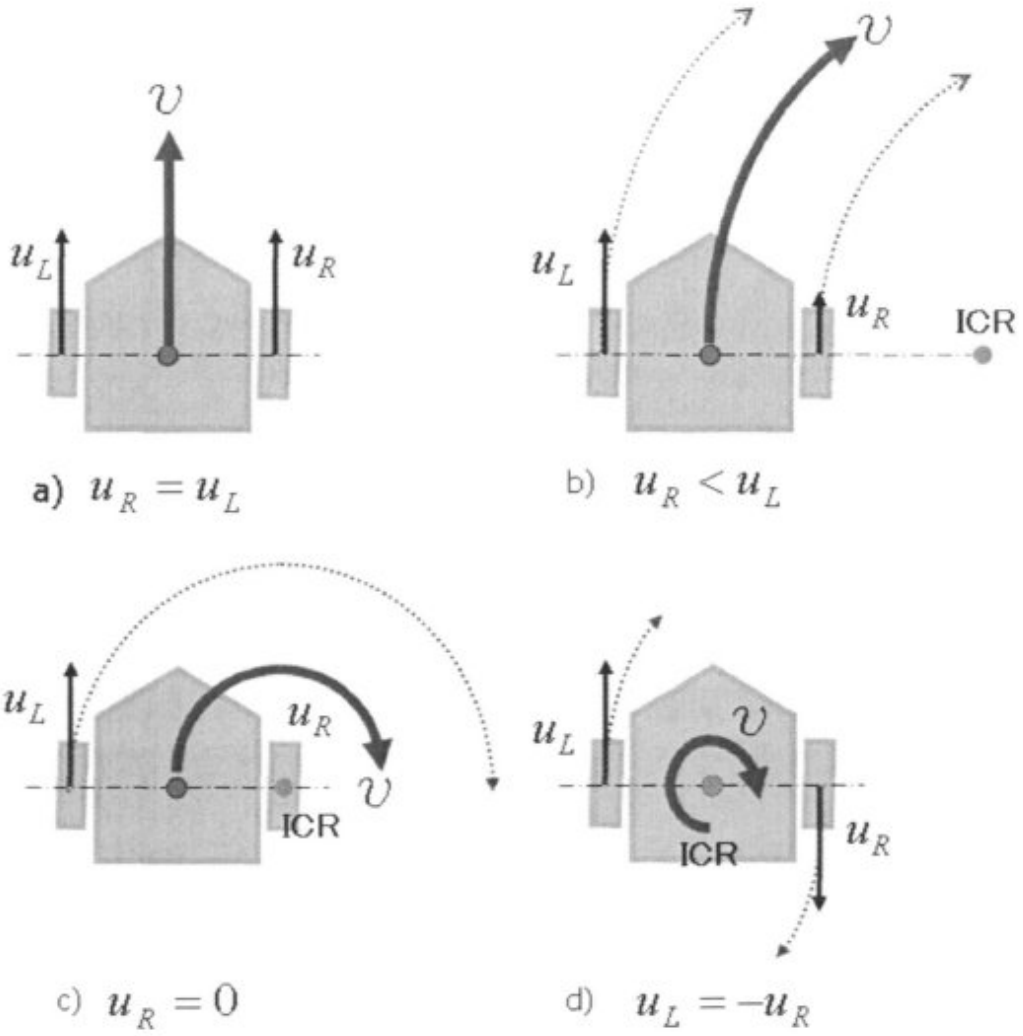
Differential Drive

Model kinematik dalam robot (terhadap gerak robot)

$$\begin{bmatrix} v_x(t) \\ v_y(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} w_l(t) \\ w_r(t) \end{bmatrix}$$

- $w_r(t)$ – angular velocity of right wheel
- $w_l(t)$ – angular velocity of left wheel

Basic Motion Control



C =ICR
(instantaneous center of rotation)

Ketika berotasi
Pusat rotasi selalu terletak
pada sumbu utama kedua roda

Jarak pusat mobot-ICR =R

Lurus … R?

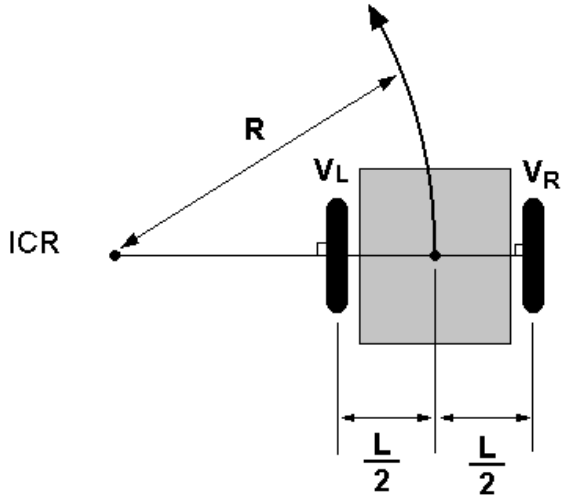
Basic Motion Control

- Instantaneous center of rotation

$$(V_R - V_L)/\ L = V_R\ /(R + \frac{L}{2})$$

$$R = \frac{L}{2}\ \frac{V_R + V_L}{V_R - V_L}$$

R : Jari2 rotasi



- Pergerakan Lurus

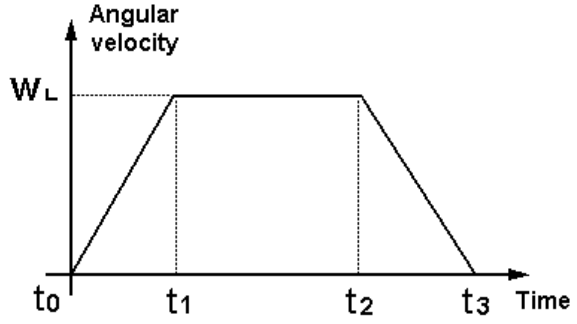
  R = Takhingga → $V_R = V_L$

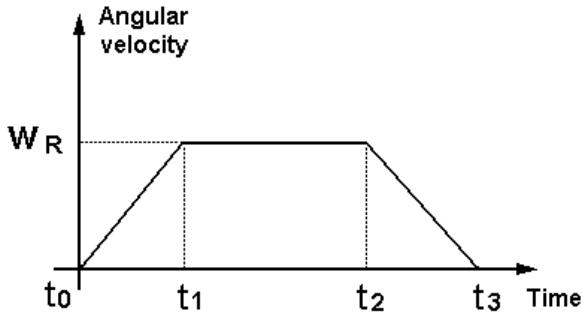
- Pergerakan Rotasi

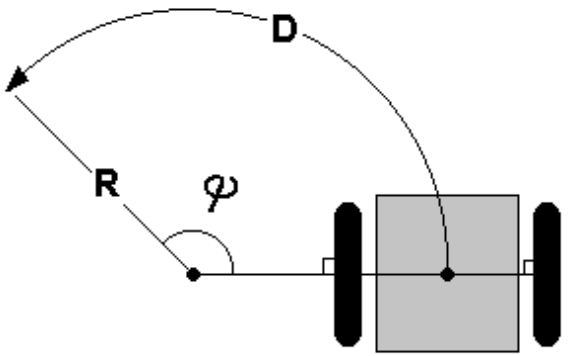
  R = 0 → $V_R = -V_L$

Basic Motion Control

- Velocity Profile







$$R = \frac{L}{2}\frac{V_R + V_L}{V_R - V_L} = \frac{L}{2}\frac{W_R + W_L}{W_R - W_L}$$

$$D = \int \frac{V_L + V_R}{2}\,dt = \frac{1}{2}\,r\frac{W_L + W_R}{2}(t_3 - t_0 + t_2 - t_1)$$

$$\varphi = \frac{D}{R} = \frac{r}{2L}(W_R - W_L)(t_3 - t_0 + t_2 - t_1)$$

R : Jari2 rotasi

D : Panjang Jalur

φ : Sudut rotasi